Chapter 5
Soil Forces and Single-Wedge Sliding Analysis

5-1. General

Chapter 4 described various loading conditions and specific loads, except for soil loads. Chapter 5 describes soil loads and explains how to use various loads in a single-wedge stability analysis. The methods presented in this chapter are intended to produce reasonably conservative estimates of soil forces acting on a structure. This manual only addresses normal soil conditions, other conditions such as swelling soils require special studies. The definitions of terms that will be used throughout this chapter are as follows:

- *Single wedge.* The single wedge is the wedge to which forces are applied, i.e., the structure itself, which is referred to as the structural wedge.

- *Applied driving forces.* Driving forces are defined as those lateral forces whose primary influence is to decrease structural stability. The side of the structure upon which these forces are applied will be called the driving side. Uplift and downdrag are also treated as applied forces.

- *Applied resisting force*s. Resisting forces are defined as those lateral forces whose primary influence is to increase structural stability. The side of the structure upon which these forces are applied will be called the resisting side. The resisting side is on the opposite side of the structural wedge from the driving side. The difference between the driving and resisting forces is transferred to the foundation by the structural wedge.

- *Reactions.* The shear and the normal force between the foundation and the base of the single wedge are reactions, which are necessary to place the structure in static equilibrium, they are not included in the applied forces.

5-2. Single-Wedge Stability Analyses

a. Basic requirements. For the single-wedge analysis, the engineer must calculate the driving and resisting soil forces, lateral water forces, and uplift and apply them to the structural wedge. The vertical drag force discussed in Appendix F may also be included, if the requirements stated in the appendix are satisfied. Using these forces and the weight of the structural wedge, the engineer can determine the magnitude, location, and slope of the resultant acting at the base of the structural wedge. The resultant will be used to evaluate sliding stability, and the location of the resultant will be used to determine the potential for partial loss of contact between the structure and the foundation materials. The resultant and its location will also be used to evaluate the bearing capacity of the foundation materials. The forces applied to the structural wedge will also be used for design of the structural elements (e.g., shears, moments and axial loads in the base and stem of a retaining wall).

b. Soil forces. Lateral soil forces acting on the single wedge should be calculated using the minimum required factor of safety against sliding to obtain the developed soil strength parameters ϕ_d and c_d. The use of developed parameters results in a force larger than the active soil force on the driving side, and a force smaller than the passive soil force on the resisting side These developed parameters shall be used in Equations 5-3 through 5-15 to calculate the lateral soil forces acting on the driving side, and used in Equations 5-16 through 5-22 to calculate the lateral soil forces acting on the resisting side. Soil forces due to seismic events are discussed in Section 5-5. If there is a significant difference between the calculated and minimum required safety factors, it may be appropriate to re-evaluate the developed soil strength parameters used to determine the soil forces. The values for the developed soil strength parameters shall be determined as:

$$\phi_d = \tan^{-1}\left(\frac{\tan\phi}{FS_s}\right) \quad and \quad c_d = \frac{c}{FS_s} \qquad (5-1, 5-2)$$

where FS_s = required factor of safety against sliding

ϕ = nominal angle of internal friction of the soil

c = nominal cohesive strength of the soil

c. *Sliding.* The resultant can be resolved into components parallel and normal to the base plane of the structural wedge. The sliding factor of safety is calculated as follows:

$$FS_s = \frac{N \tan \phi + c\,L}{\mathrm{T}} \tag{5-3}$$

where: N = the component of the resultant normal to the base

T = the component of the resultant parallel to the base

L = length of base in compression

If the safety factor is equal to or greater than the required safety factor, the sliding stability criterion is satisfied. Note that this calculated safety factor might not be equal to the minimum factor used to determine developed soil strength parameters. If there is a significant difference between the calculated and minimum required safety factors, it may be appropriate to re-evaluate the developed soil strength parameters used to determine the soil forces. The resultant location shall be used to determine the length of the base that is in compression, and cohesion shall not be effective on that part of the base that is not in compression. If there is any loss of contact, uplift forces should be re-evaluated. However, for seismic events, cyclic loading periods are so short that the time is not sufficient for uplift pressures to change, therefore, uplift should not be increased due to loss of contact for seismic loads.

5-3. Soil Pressures and Forces

a. Active soil pressures.

(1) Cohesionless backfill. Cohesionless materials such as clean sand are the recommended backfill for most structures. Large-scale tests (Terzaghi 1934; Tschebatarioff 1949; Matsuo, Kenmochi, and Yagi 1978) with cohesionless backfills have shown that lateral pressures are highly dependent on the magnitude and direction of wall movement. The minimum lateral pressure condition, or active soil pressure, develops when a structure rotates about its base and away from the backfill an amount on the order of 0.001 to 0.005 radians. As the structure moves, horizontal stresses in the soil are reduced, and vertical stresses due to backfill weight are resisted by increasing shear stresses until shear failure is imminent (Figures 5-1 and 5-2).

(2) Cohesive backfill. For situations where cohesive backfill is unavoidable, solutions are included herein for soil pressures involving both frictional and cohesive soil strength parameters (ϕ and c). Where cohesive backfill is used, two analyses (short-term and long-term) are usually required in order to model conditions that may arise during the life of the structure. Short-term analyses model conditions prevailing before pore water pressure dissipation occurs, such as the end-of-construction condition. Unconsolidated-undrained test parameters, which yield a relatively high cohesion value and a low or zero friction value, are appropriate for short-term analyses. Long-term analyses model conditions prevailing after shear-induced pore pressures have dissipated. For long-term analyses, consolidated-drained test parameters are appropriate. These tests usually yield a relatively high value for internal friction and a low or zero value for cohesion.

b. Passive soil pressures. If a structure is moved toward the backfill, lateral soil pressures increase and shear stresses reverse direction, first decreasing and then increasing to a maximum at failure (Figure 5-4). Full development of passive pressure requires much larger structure rotations than those required for the active case, as much as 0.02 to 0.2 radians (Figures 5-1 and 5-2). However, the rotation required to develop one-half of the passive pressure is significantly less, as little as 0.005 radians. The designer must be certain that soil on the resisting side of any structure will always remain in place and not be excavated or eroded before its effect is included in the stability analyses.

c. At-rest soil pressure. If no structural movement occurs, then the at-rest condition exists.

d. Design soil pressures - driving side. In practice, the active and passive soil pressure conditions seldom exist. Hydraulic structures are designed using conservative criteria that results in relatively stiff structures. Structures founded on rock or stiff soils usually do not yield sufficiently to develop active pressures. Even for foundations capable of yielding, experiments with granular backfill (Matsuo, Kenmochi, and Yagi 1978) indicate that following initial yield and development of active pressures, lateral pressures may in time return to greater values. Another reference (Casagrande 1973) states that the gradual buildup of the backfill in compacted lifts produces greater-than-active pressures as do long-term effects from vibrations, water level fluctuations, and temperature changes. For these reasons and because large rotations are required for the development of passive pressures, soil pressures on both the driving side and the resisting side of the single wedge will be estimated by using the developed soil strength parameters, as defined in paragraph 5-2. These parameters are then used to calculate the *equivalent-fluid* soil pressure coefficients (K).

(1) General wedge method for equivalent fluid pressure coefficients. Lateral soil forces are assumed to act parallel to the top surface of driving side wedges when the surface slopes downward toward the structure. Equivalent fluid-pressure coefficients are calculated as follows:

$$\alpha = \tan^{-1}\left(\frac{C_1 + \sqrt{C_1^2 + 4\,C_2}}{2}\right) \tag{5-4}$$

where α = the critical slip plane angle for the soil wedge (see Appendix E for a derivation of α)

$$C_2 = \frac{t + \left(\dfrac{2V}{\gamma(h^2 - d_c^2)}\right)(1 + \tan^2\phi_d)\tan^2\beta + \left(\dfrac{2\,c_d}{\gamma(h + d_c)}\right)r}{A}$$

$$A = \tan\phi_d + \tan\delta - \left(\frac{2V}{\gamma(h^2 - d_c^2)}\right)(1 + \tan^2\phi_d) + \left(\frac{2\,c_d}{\gamma\,(h + d_c)}\right)r$$

$r = 1 - \tan\delta \tan\phi_d - \tan\beta\,(\tan\delta + \tan\phi_d)$

$s = \tan\beta + \tan\phi_d + \tan\delta\,(1 - \tan\beta \tan\phi_d)$

$t = \tan\phi_d - \tan\beta - (\tan\delta + \tan\beta)\tan^2\phi_d$

ϕ = soil internal friction parameter

ϕ_d = developed internal friction parameter

c = soil cohesion parameter

c_d = developed cohesion parameter

β = top surface slope angle, positive when slope is upward when moving away from the structure (When the top surface of the backfill is broken, solutions for α may be obtained by using analogous positive and negative strip surcharges.)

δ = wall friction angle. When β is positive, $\delta = \beta$. When β is zero or negative $\delta = 0$. Vertical shear (drag), as discussed in Appendix F, shall not be used to calculate the value of α or equivalent fluid soil pressure coefficients. However, drag may be used in addition to lateral soil pressures when the requirements of Appendix F are satisfied.

γ = average unit weight of soil (moist weight above water table, buoyant weight below)

V = strip surcharge

h = height of vertical face of soil wedge

d_c = depth of cohesion crack in soil (should always be assumed filled with water when calculating lateral forces)

The equivalent fluid-pressure coefficients are:

$$K = \frac{1 - \tan\phi_d \cot\alpha}{\cos\delta\,[(1 - \tan\delta\tan\phi_d) + (\tan\phi_d + \tan\delta)\tan\alpha]}$$

and for soils that possess cohesive properties as well as internal friction:

$$K_c = \frac{1}{2\cos^2\alpha\,(\tan\alpha - \tan\beta)\,[1 - \tan\delta\tan\phi_d + (\tan\delta + \tan\phi_d)\,tanalpha\,]}$$

The equation for the depth of a cohesive crack is:

$$d_c = \frac{2\,K_c\,c}{K\gamma\left(\dfrac{\tan\alpha}{\tan\alpha - \tan\beta}\right)}$$

And the total lateral soil force is calculated as:

$$P = \frac{1}{2}K\gamma\left(\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right)(h - d_c)^2 + K\,V\tan\alpha$$

Examples are presented in Appendix D.

When any of the variables in the above equations are not present in a particular problem, they are set equal to zero, thereby simplifying the equations. Figure 5-3 illustrates a wedge containing cohesionless soil, showing the methods used to calculate the lateral force, and the soil pressure at any point on the vertical face of the wedge. Figure 5-4 shows similar information for a wedge consisting of a cohesive soil. Figure 5-5 shows the method used to determine the pressure distribution for a strip surcharge (a line load V).

When ß is greater than ϕ_d a solution for α cannot be obtained from Equation 5-4 because the number under the radical will be negative, making the square root indeterminate. However, when ß is equal to ϕ_d, Equation 5-4 will give a value for α equal to ß and ϕ_d. When $\alpha = \phi_d =$ ß, the total lateral soil force, for a granular soil not supporting a strip surcharge, is:

$$P = \frac{1}{2} \gamma h^2 \cos \phi_d$$

This equation gives the maximum driving side lateral soil force that can occur and should be used when ß is equal to or greater than ϕ_d.

(2) Equivalent fluid pressure coefficients from Coulomb's equation. Coulomb's equation may be used to calculate the equivalent fluid-pressure coefficient when the surface of the backfill wedge is planar and unbroken, if certain conditions are met. These conditions are:

- When backfill has a sloping top surface. There can be only one soil material. The water-table must be either completely above or completely below the backfill. The backfill must be cohesionless. Surcharges must be uniformly distributed and cover the entire top surface of the backfill wedge.

- When backfill has a horizontal top surface. There may be more than one soil material, if the top surface of all soil layers are horizontal. The water-table may lie within the backfill. The soil may be either cohesive or cohesionless. Surcharges must be uniformly distributed and cover the entire top surface of the backfill wedge.

When any of the above conditions are not applicable, the equivalent fluid pressure coefficient determined by the wedge method shall be used. The equation for the equivalent fluid-pressure coefficient, using the Coulomb Equation is:

$$K = \frac{\cos^2 \phi_d}{\cos \delta \left[1 + \sqrt{\frac{\sin(\phi_d + \delta) \sin(\phi_d - \beta)}{\cos \delta \cos \beta}} \right]^2}$$

and the total lateral soil force is:

$$P = \frac{1}{2} K \gamma h^2$$

(3) Equivalent fluid pressure coefficients for simple conditions. When the top surface of the wedge is horizontal, planar, supports a uniform surcharge covering the entire top surface, and the soil possesses cohesive strength as well as internal friction, the equivalent fluid pressure coefficients in the preceding equations reduce to the following simple expressions:

$$K = \frac{1 - \sin \phi_d}{1 + \sin \phi_d} = \tan^2 (45° - \frac{\phi_d}{2})$$

$$K_c = \sqrt{K}$$

$$d_c = \frac{2 c_d}{\gamma \sqrt{K}}$$

e. Design soil pressures - resisting side. Developed soil pressures on the resisting side may also be calculated using the developed soil strength parameters. In this manual, soil pressures and forces on the resisting side are generally assumed to act horizontally (wall friction angle δ = 0). The equivalent fluid-pressure coefficient for soil pressures on the resisting side is calculated as follows:

$$\alpha = \tan^{-1}\left[\frac{-C_1 + \sqrt{C_1^2 + 4C_2}}{2}\right]$$

$$C_1 = \frac{2\tan^2\phi_d - \frac{4V}{\gamma h^2}[\tan\beta(1+\tan^2\phi_d)] + \frac{4c_d}{\gamma h}(\tan\phi_d - \tan\beta)}{A}$$

$$C_2 = \frac{\tan\phi_d(1+\tan\phi_d\tan\beta) + \tan\beta + \frac{2c_d(1+\tan\phi_d\tan\beta)}{\gamma h} - \frac{2V\tan^2\beta(1+\tan^2\phi_d)}{\gamma h^2}}{A}$$

where

$$A = \tan\phi_d + \frac{2c_d(1+\tan\phi_d\tan\beta)}{\gamma h} + \frac{2V(1+\tan^2\phi_d)}{\gamma h^2}$$

Then the equivalent fluid-pressure coefficients for resisting side pressures are:

$$K_P = \frac{1+\tan\phi_d\cot\alpha}{1+\tan\beta\tan\phi_d - (\tan\phi_d - \tan\beta)\tan\alpha}$$

and when cohesion is present:

$$K_{cP} = \frac{1}{2\cos^2\alpha(\tan\alpha - \tan\beta)[1+\tan\beta\tan\phi_d - (\tan\phi_d - \tan\beta)\,tanalpha]}$$

The developed soil force on the resisting side is then:

$$P_P = \frac{1}{2}K_P\gamma\left(\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right)h^2 + 2K_{cP}c_d h + K_P V\tan\alpha$$

Since cohesive cracking will not occur on the resisting side, the term for the depth of cohesive cracking (d_c) is not applicable to these equations.

5-4. Soil Pressures with Water Table Within or Above Top of Backfill Wedge

Pressures and forces due to soil and water must be calculated separately, since wall friction (δ) is not applicable to water pressure and the equivalent fluid pressure coefficients (K and K_c) are for calculating lateral soil pressure only. K for water is always equal to one. However, the effective unit weight of soil below the water table is affected by the uplift due to water. In lateral soil-pressure calculations, the moist unit weight of soil is used above the water table, and the buoyant unit weight is used below the water table. When calculating lateral water pressure and uplift, the effect of seepage (if it occurs) must be considered. Lateral soil pressures and forces are calculated as shown below, and as illustrated in Figure 5-6. These must be added to the lateral water pressure.

$$P = \frac{1}{2} p_s (h - h_s) + \frac{1}{2} (p_s + p) h_s$$

Where: P = the total lateral soil force,

and $$p_s = K \gamma_m \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} \right) (h - h_s) \text{ = lateral soil pressure at water table}$$

$$p = K \left[\gamma_m h \left(\frac{\tan \alpha}{\tan \alpha - \tan \beta} \right) - (\gamma_m - \gamma_b) h_s \right] \text{ = lateral soil pressure at bottom of soil wedge}$$

γ_m = moist unit weight of soil, use above water table

γ_b = buoyant unit of weight of soil, use below water table

h = height of vertical face of soil wedge

h_s = height of water table above bottom of wedge

5-5. Earthquake Inertial Forces on Structures

The seismic response of structures with backfill is complicated because in addition to the interaction with the foundation, the structure is also subjected to the dynamic soil pressures induced by ground shaking. Dynamic backfill pressures are related to the relative movement between the soil and the structure and the stiffness of the backfill. Behavior of the structure may be controlled by rocking and/or translation in response to earthquake shaking. The type of response will correlate to different distributions of backfill pressure acting against the wall. The appropriate method for analyzing the backfill pressure may be categorized according to the expected movement of the backfill and structure during seismic events.

a. Behavior of Backfills.

(1) Non-Yielding Backfills. For low intensity ground motions the backfill material may respond within the range of linear elastic deformations. Walls with non-yielding backfills can be expected to have dynamic soil pressures greater than those predicted by the Mononobe-Okabe method. The dynamic soil pressures and associated forces in the backfill may be analyzed as an elastic response using Wood's method as described in ITL 92-11 (Ebeling and Morrision, 1992). A reasonable estimate for determining the additional lateral seismic soil force against a soil retaining structure, for non-yielding backfill conditions, can be determined as

$$F_{sr} = \gamma h^2 k_h$$

where: F_{sr} = Lateral seismic force representing dynamic soil pressure effects

γ = Unit weight of soil. Use moist or submerged unit weight when all soil is above or below the water table. For partially submerged soils the unit weight shall be proportioned by a weighted average.

h = Height of backfill

k_h = Effective peak ground acceleration, expressed as a decimal fraction of the acceleration of gravity

The seismic component of the total soil force F_{sr} is assumed to act at a distance of 0.63 h above the base of the wall. This force must be combined with the structure lateral inertial force, and if water is present, hydrodynamic seismic forces to obtain the total seismic force on the wall. Evaluation of a wall with non-yielding backfill for the aforementioned seismic forces is illustrated by Example 32 of ITL-92-11 (Ebeling and Morrison, 1992). The various seismic forces described above must be combined with static soil pressure forces and static water pressure forces to get the total force on the wall. Soil retaining structures not meeting stability criteria using the preliminary screening method should be evaluated using refined analysis techniques described in ITL-92-11 (Ebeling and Morrison, 1992).

(2) Yielding Backfills. The relative motion of the structure and backfill material may be large enough to induce a limit or failure state in the soil. This condition may be modeled by the Mononobe-Okabe method (Mononobe and Matsou (1929), and Okabe (1926)), in which a wedge of soil bounded by the structure and an assumed failure plane are considered to move as a rigid body with the same horizontal acceleration. The dynamic soil pressures using this method are described in Appendix G.

(3) Partially Yielding Backfills. The intermediate condition in which the backfill soil undergoes limited nonlinear deformations corresponds to the shear strength of the soil being partially mobilized. The dynamic backfill pressures may be estimated using an idealized constant parameter, SDOF model of a semi-infinite uniform soil layer (Veletsos and Younan 1994) or a frequency-independent, lumped parameter, MDOF system. The dynamic pressures for an irregular backfill may be analyzed using a soil-structure-interaction model such as FLUSH (Lysmer et al. 1975). The wall is usually modeled with 2-D elements. The foundation rock is represented by 2-D plane-strain elements with an appropriate modulus, Poisson's ratio, and unit weight. Transmitting boundaries in the form of dashpots are introduced at the sides of the foundation rock to account for the material nonlinear behavior with depth. The shear modulus and soil damping vary with the level of shearing strain, and this nonlinear behavior is usually approximated by an equivalent linear method. The boundary conditions for the backfill may also be represented by dashpots. Hydrodynamic pressures exerted on the wall are computed using the Westergaard formula.

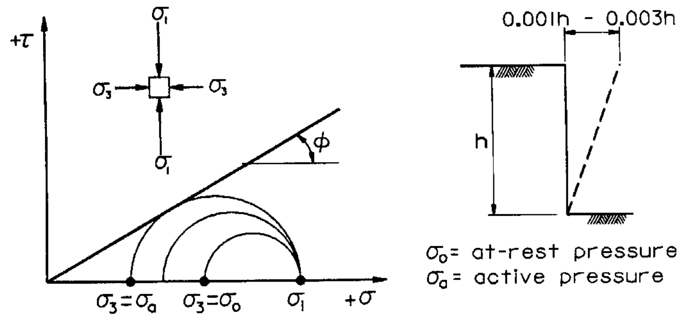
b. Simplified Wedge Method. A seismic coefficient method may be used to estimate the backfill and wall inertial forces acting on a single wedge. Theoretically, a structure (wedge) may behave as a rigid body that is fully constrained along its base and sides by the ground, so all parts of the wedge would be uniformly affected by accelerations which are identical to the time history of the ground motions. Therefore it would be appropriate to use a seismic coefficient equal to the peak ground acceleration for stability analysis of short, stiff structures. However, field and test data show that most structures do not behave as a rigid body, but respond as a deformable body subjected to effective ground motions. Thus the magnitude of the accelerations in a deformable wall may be different than those at the ground surface, depending on the natural period and damping characteristics of the structure and the shaking characteristics of the ground motions. Furthermore the maximum acceleration will only affect the structure for a short interval of time, and the inertia forces will not be equivalent to those of an equal static force which would act for an unlimited time, so the deformations resulting from the maximum acceleration will be smaller. Design or evaluation of structures for zero relative displacement under peak ground accelerations is unrealistic, so the seismic stability analysis should be based on a seismic coefficient which recognizes that an acceptably small amount of lateral displacement will likely occur during a major earthquake. Experience has shown that a seismic coefficient equal to 2/3 of the peak ground acceleration is a reasonable estimate for many hydraulic structures. For partially yielding backfill, the strength mobilization factor should be equal to the reciprocal of the minimum required sliding safety factor for that load case. More information about the simplified wedge method is included in appendix G.

5-6. Mandatory Requirements

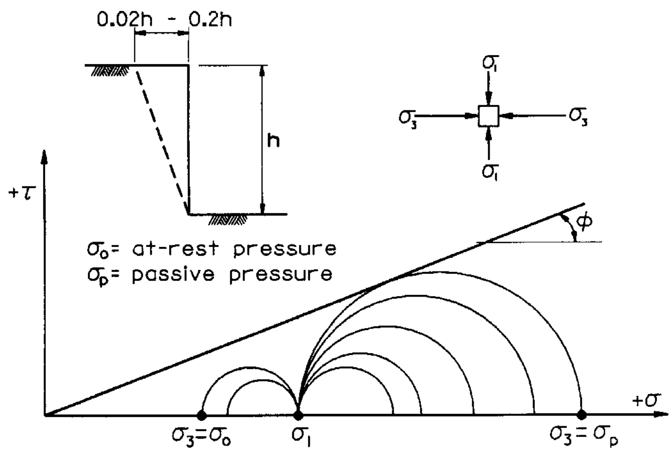
For a general discussion on mandatory requirements see Paragraph 1-5. As stated in that paragraph, certain requirements within this manual are mandatory. The following are mandatory for Chapter 5.

a. Developed Soil Strength Parameters. Lateral soil forces acting on a single wedge shall be determined using developed soil strength parameters as described in paragraph 5-2.b.

b. Sliding Factor of Safety. The factor of safety for sliding shall be calculated as defined in paragraph 5-2.c.



a. Development of active earth pressure



b. Development of passive earth pressure

Figure 5-1 Soil Pressures for Cohesionless Material

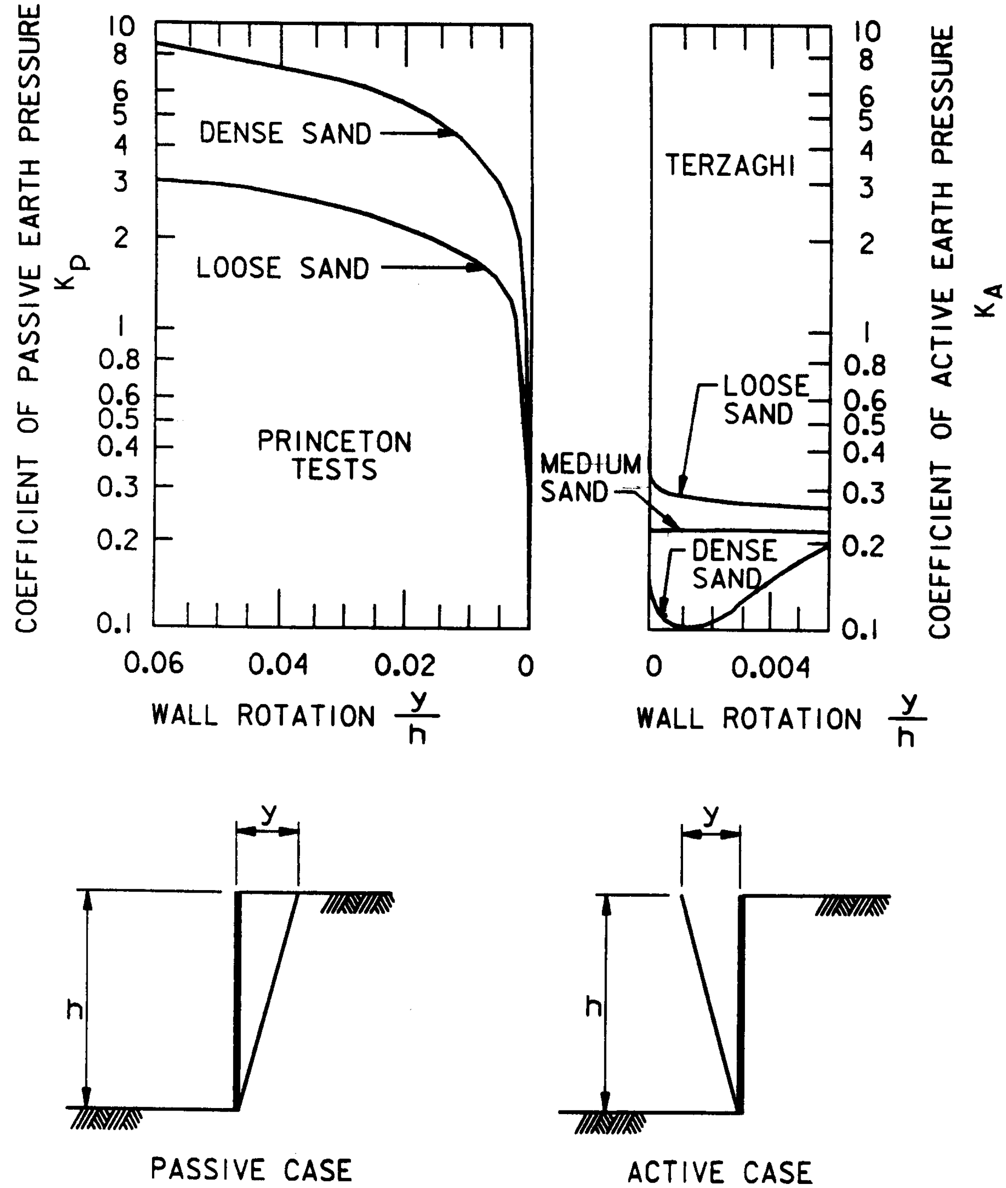
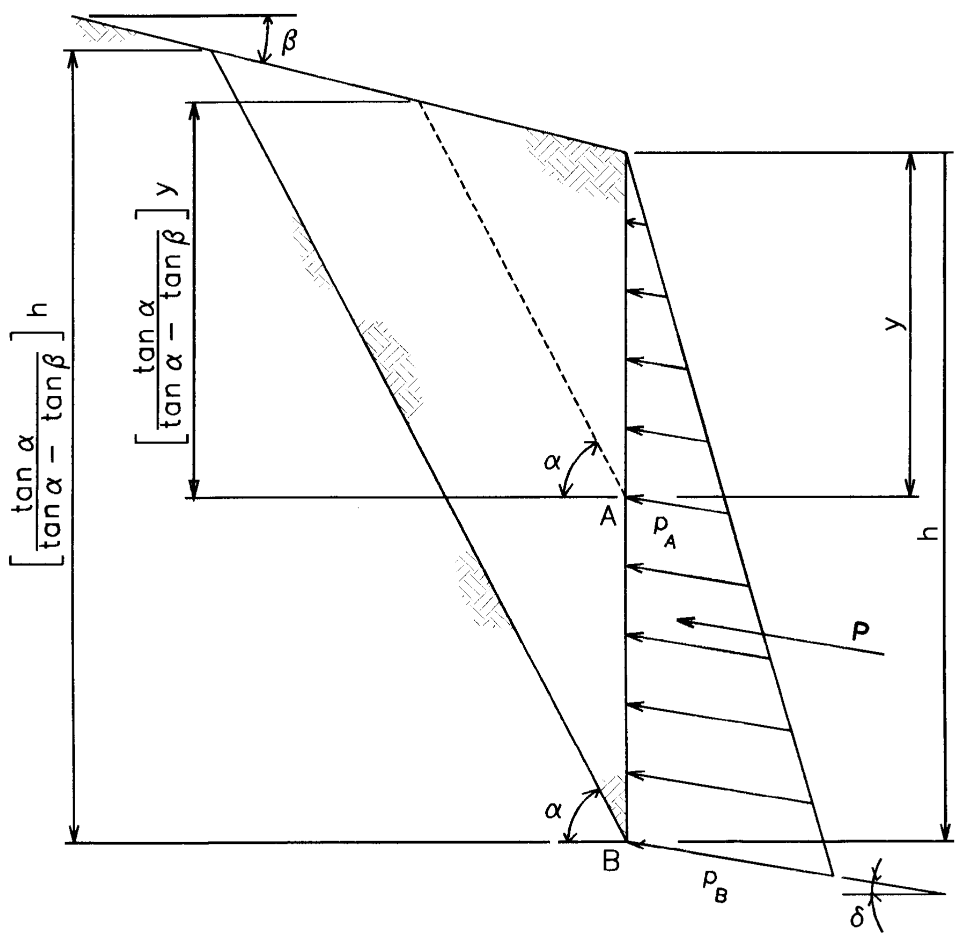


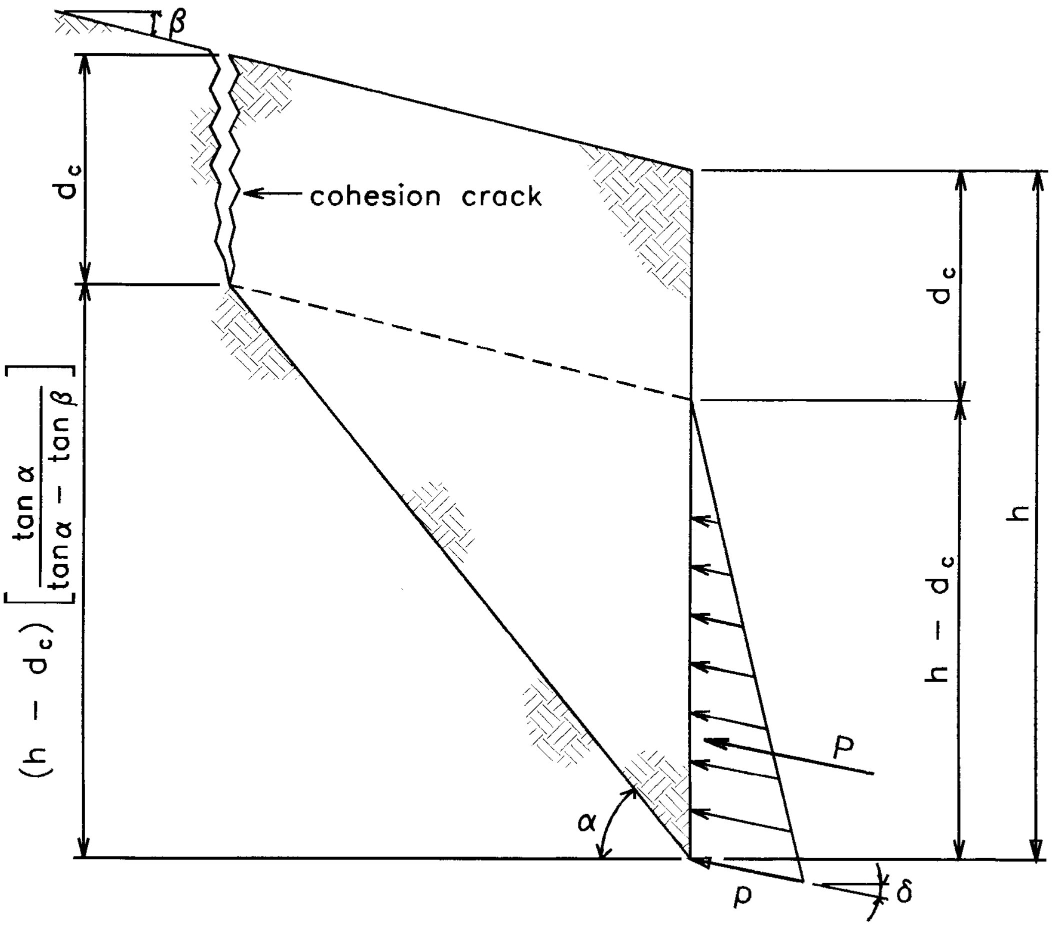
Figure 5-2 Soil Pressures versus Wall Movements



$$p_A = K\gamma \left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right] y \qquad p_B = K\gamma \left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right] h$$

$$P = 1/2\, K\gamma \left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right] h^2$$

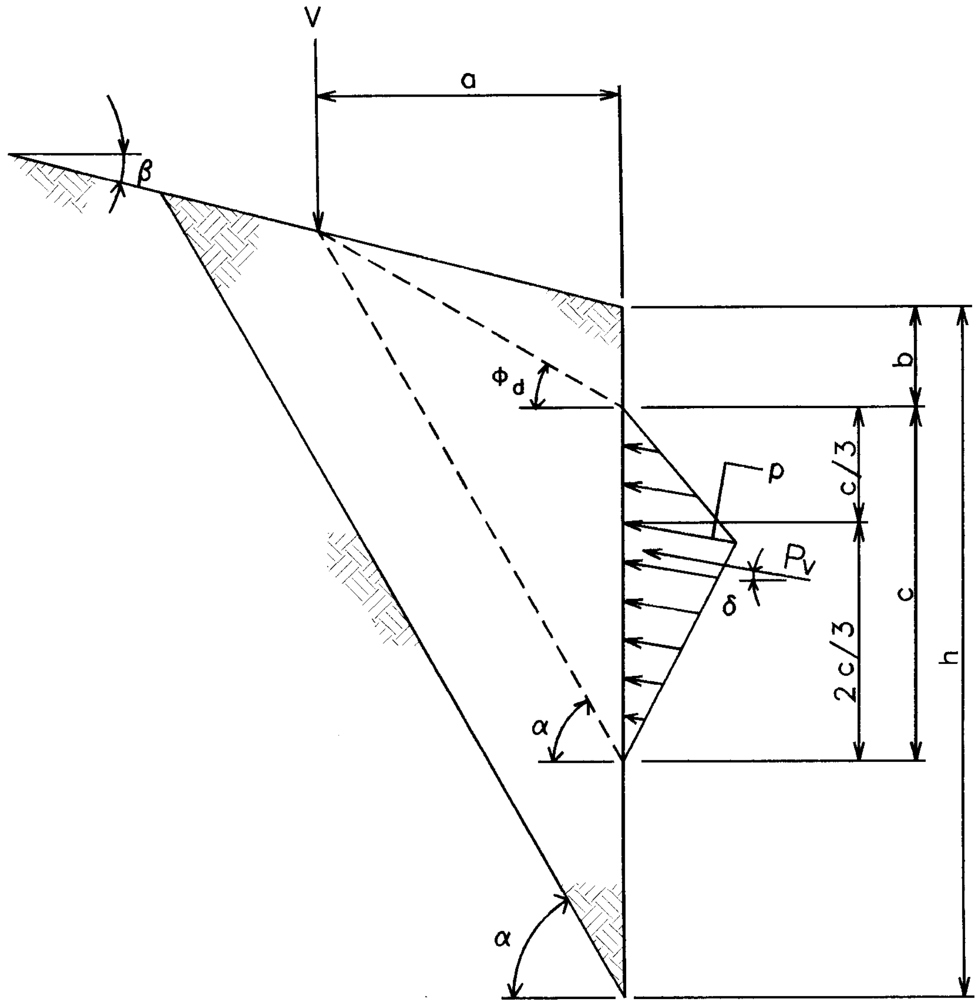
Figure 5-3 Calculation of Soil Pressures for Cohesionless Material



$$p = K\gamma\left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right](h - d_c)$$

$$P = 1/2\ K\gamma\left[\frac{\tan\alpha}{\tan\alpha - \tan\beta}\right](h - d_c)^2$$

Figure 5-4 Calculation of Soil Pressures for Cohesive Material



$$b = a\,(\tan \Phi_d - \tan \beta)$$

$$c = a\,(\tan \alpha - \tan \Phi_d)$$

$$P_v = K\,V\,\tan \alpha$$

$$p = \frac{2P_v}{c}$$

Figure 5-5 Lateral soil pressures due to strip surcharge loads

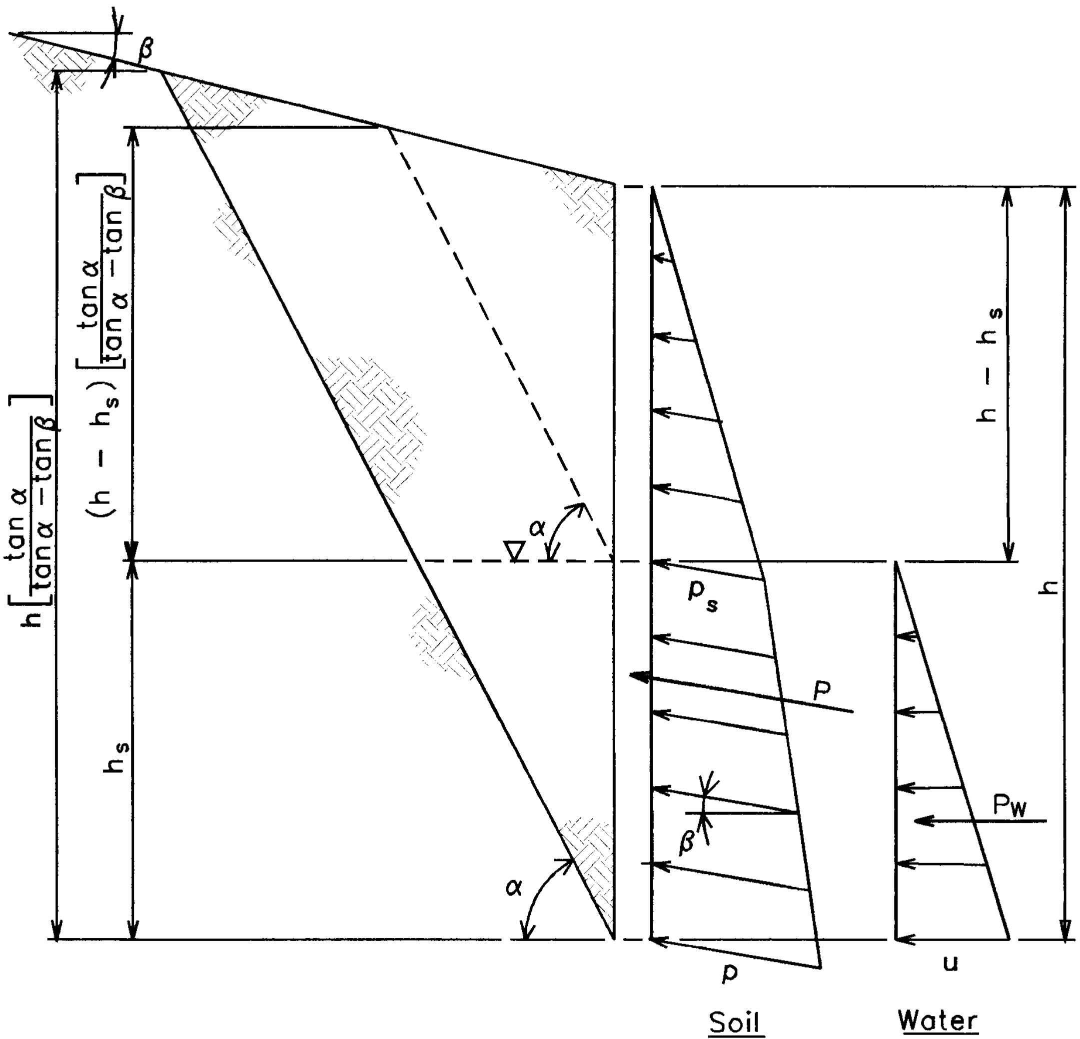


Figure 5-6 Lateral Soil Pressures with Water Table